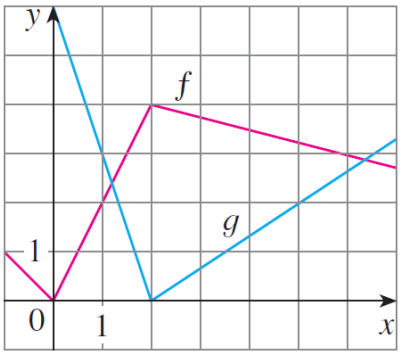
Exercise 65

If f and g are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find each derivative, if it exists. If it does not exist, explain why.

(a) $u'(1)$ (b) $v'(1)$ (c) $w'(1)$



Solution

Take the derivative of $u(x)$.

$$u'(x) = f'(g(x)) \cdot g'(x)$$

Evaluate it at $x = 1$.

$$\begin{aligned} u'(1) &= f'(g(1)) \cdot g'(1) \\ &= f'(3) \cdot (-3) \\ &= \left(-\frac{1}{4}\right) \cdot (-3) \\ &= \frac{3}{4} \end{aligned}$$

Take the derivative of $v(x)$.

$$v'(x) = g'(f(x)) \cdot f'(x)$$

Evaluate it at $x = 1$.

$$\begin{aligned} v'(1) &= g'(f(1)) \cdot f'(1) \\ &= g'(2) \cdot (2) \end{aligned}$$

There's a kink in the graph of g at $x = 2$, so $g'(2)$ is undefined. $v'(1)$ is undefined, too, as a result.

Take the derivative of $w(x)$.

$$w'(x) = g'(g(x)) \cdot g'(x)$$

Evaluate it at $x = 1$.

$$\begin{aligned} w'(1) &= g'(g(1)) \cdot g'(1) \\ &= g'(3) \cdot (-3) \\ &= \left(\frac{2}{3}\right)(-3) \\ &= -2 \end{aligned}$$